

*Methods of determining the Tilt of Photographic Plates.*

By R. J. Pocock, B.A., B.Sc.

Last year, when erecting the Hyderabad astrographic telescope, the question of adjusting the tilt of the plates presented some difficulty. The photographic end of the telescope was made in the Observatory workshop, and so arranged that the plate rests on three steel screws, which I had lacquered to prevent rusting.

To prevent accidental movement of these screws, they are kept firm by three lateral tightening screws. The usual method of adjusting these screws is to use a small collimator attached to the back of a plate. This method was used here, fine cross-wires being stretched across the object-glass without touching the lens, and the screws adjusted so that these cross-wires, the centre of the plate, and the cross-wires in the collimator were in a line for all orientations of the collimator. The collimator used is a rather cumbrous instrument constructed in the workshop, and consequently this method did not seem altogether satisfactory—beyond assuring that the tilt was small. One objection to this method which may have escaped notice elsewhere is the possibility that the plate is not uniformly thick. Since the collimator is attached to the back of the plate, any such want of uniformity will introduce an apparent tilt where none really exists.

When testing the Oxford astrographic telescope for tilt about a year and a half ago, a thin plate (plate-glass) was used, and on measuring with calipers at the points where it was supported in the telescope the differences in thickness were found to be such as to produce an apparent tilt about equal to that usually found in such instruments.\*

The adjustment of the Hyderabad telescope not being quite satisfactorily made, it seemed desirable, if possible, to determine the tilt from the measures themselves.

*Tilt determined from Star Counts.*

In the first place, the simple method of counting suggests itself. The star density varies in different parts of the plate: in the absence of any other cause for variation the density should be symmetrical about the plate centre—this will not be the case if the plate is tilted.

Let  $r_0$  be the distance from the plate centre of the position of best focus for an untilted plate,  $r_1, r_2$  the distances measured along (say) the  $x$ -axis for a tilted plate with the same centre. Let  $f$  be the focal length of the telescope,  $k$  the distance between the plate centre and the object-glass,  $\psi$  the tilt of the plate. Then we have the following geometrical relations:—

$$\begin{aligned} f^2 &= r_0^2 + k^2 \\ r_0^2 &= r_1^2 - 2r_1 \cdot k \sin \psi = r_2^2 + 2r_2 \cdot k \sin \psi \\ r_1 - r_2 &= 2k \sin \psi \\ r_0^2 &= r_1 \cdot r_2. \end{aligned}$$

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\* See note, p. 665.

The perpendicular from the centre of the object-glass on the tilted plate meets it at a point distant  $\frac{r_1 - r_2}{2} = k \cdot \sin \psi$  from the centre.

Professor Turner has given an approximate formula\* for the density at any point of a plate, viz.

$$D = a \{ 1 \sim c(r^2 - r_0^2) \},$$

where  $a, c$  are certain constants. This formula agrees well with observation for all but large values of  $r$ .

Consider a circle of radius 13 réseaux intervals ( $= b$  say) with centre at the plate centre: we may neglect the corners of the plate beyond this circle, as they contain few stars. We can integrate the above formula over a quadrant of this circle, and thus find the density  $D$  on one half the plate. This is on the assumption that for practical purposes we may replace one eccentric circle of best focus by two concentric semicircles.

$$\begin{aligned} D_1 &= 2 \int_0^{\pi/2} \int_0^b a [1 \sim c(r^2 - r_0^2)] r \cdot dr \cdot d\theta \\ &= \pi a \left\{ \frac{b^2}{2} + \frac{cb^4}{4} - \frac{bcr_1^2}{2} + \frac{cr_1^4}{2} \right\}. \end{aligned}$$

If we assume  $r_1 = 30'$ ,  $r_2 = 35'$  with the values  $b = 65'$ ,  $c = 10^{-4}$ , we find

$$\frac{D_1 - D_2}{D_1} = 0.13.$$

In this case  $\frac{r_2 - r_1}{2} = k \sin \psi = 2\frac{1}{2}' = 0.5$  réseau intervals.

For a telescope of focal length  $3^m.44$  adjusted so that the circle of best focus has a radius of about  $32'$ , an excess of stars amounting to 1.3 per cent. may be due to a tilt of about  $2'$ .

If we could be sure that the star density was uniform over the region covered by the plate, this method would form a very delicate test of the tilt of a plate.

#### *Application to the Oxford Plates.*

Detailed counts of some Oxford plates are given in *Monthly Notices*, March 1908, p. 399.

If we consider the regions  $a$  to  $g$  of Table II. of that paper, these are practically the same as the circle radius 13 considered above.

Denoting the sum of  $XY + Xy$  for the regions  $a$  to  $g$  by  $X$ , etc., we find

$$\begin{aligned} X &= 2212, & Y &= 2313, & x &= 2135, & y &= 2034 \\ X - x &= 77, & Y - y &= 279. \end{aligned}$$

\* *M.N., R.A.S.*, lxii. p. 434.

The plates are nearly all in the first three hours of R.A., and there is no sensible systematic increase in the number of stars with R.A.; we may therefore consider the difference  $X - x$  as entirely due to instrumental causes. On the other hand, the difference  $Y - y$  is probably in part at any rate due to a systematic difference in the number of stars depending on the declination. The different numbers of stars in the different volumes of the catalogue might furnish some indication of the magnitude of this difference, but it would hardly be reliable, in view of the great differences in sensitiveness of plates and in the amount of revision work done for some volumes. We shall consider, therefore, only the difference  $X - x$ .

We have

$$\frac{D_1 - D_2}{D_1} = \frac{77}{2212} = \cdot 035.$$

According to the above formula this would give a tilt of 5' or 6', not an improbable amount. The excess, of course, may be due to a combination of tilt of object-glass and tilt of plate.

### *The Hyderabad Plates.*

The Hyderabad instrument, which is fitted with an 8-inch Cooke photovisual lens, does not appear to be well focussed. The instrument was focussed in 1914 December. It appeared then, from counts of stars on several plates taken for the purpose, that the density was very uniform over the plate: the position of best focus appeared to be near the regions  $f, g$ , as in the case of the Oxford plates. Since then, however, there seems to have been a considerable change in the focus, which may be partly due to the great difference in temperature between December and April. The position of best focus now appears to be the centre of the plate, and the density falls off rapidly towards the edge.

The following table represents the density in different regions of the plate:—

	<i>a.</i>	<i>b.</i>	<i>c.</i>	<i>d.</i>	<i>e.</i>	<i>f.</i>	<i>g.</i>	<i>h.</i>	<i>i.</i>	<i>k.</i>
<i>xy</i>	27	20	21	19	20	20	16	15	15	14
<i>Xy</i>	24	25	20	19	21	19	17	15	12	12
<i>xY</i>	18	24	21	23	21	20	19	18	18	15
<i>XY</i>	24	22	21	21	21	20	21	17	15	13
Total	93	91	83	82	83	79	73	65	60	54
1914 Dec.	51	53	67	49	53	59	56	48	48	

It is possible that as at present adjusted the plate does not cut the surface of best focus at all, in which case the above formula may not be applicable; but it may be noted that, except for bright stars near the edge of the plate, the images do not appear sensibly out of focus. The numbers in the line 1914 December represent the density for intermediate regions at that date, but depend on fewer stars than the numbers above. The results for 1914 December

depend on counts of images on the plates; the later results depend on counts of actual measures and may be affected by a natural tendency to measure fewer stars near the edges of the plates where the images are bad—and in the case of the Hyderabad plates where the réseau lines are not quite so good. Colour is lent to this idea by the fact that the plate constants show no evidence of sensible change of focus. Taking the mean values of A, E for plates taken last December for the plates considered above and for still later plates, we find

$$\begin{array}{lll} A & -0.1762, & -0.1762, & -0.1756 \\ E & -0.1777, & -0.1784, & -0.1768 \end{array}$$

The difference  $A - E$  is due to refraction; the mean refraction for plates on the meridian gives  $A - E = +0.0014$  in Zone  $-17^\circ$ .

We have

$$\begin{array}{llll} X = 4694, & Y = 4879, & x = 4661, & y = 4476, \\ & X - x = 33, & Y - y = 403. \end{array}$$

We can estimate the systematic difference  $X - x$  from the plates themselves: thus, the first seven plates average 1000 stars each, the remaining eight average 700. The average increase between two plates is therefore 40. The average difference between two half plates is therefore 10, the earlier half being the larger.

Reducing this number in the ratio of the number of stars from  $a$  to  $g$  to the total number of stars on the plates, we obtain

$$X - x = -\left(\frac{10 \times 9355}{12598}\right) \times 15 = -111.$$

Therefore the difference which may be due to tilt is  $X - x = 144$

$$\frac{D_1 - D_2}{D_1} = \frac{144}{4694} = 0.031.$$

If in reality the best focus is about  $r = 32'$ , this would mean a tilt of about  $5'$ , equivalent to a displacement of the centre a little more than 1 réseau interval.

No attempt has been made to determine the tilt of the object-glass by means of a collimator. The lens is only an 8-inch, and with the large collimator we have it would be impossible to test it far from the centre, so that the result would have little value and the risk of damaging the lens would be too great.

*The Plate Constants obtained on the Assumption of a Tilted Plate.*

Finally, we may determine the tilt of a plate by computing the actual point at which the perpendicular from the object-glass meets the plate. We can do this if we include in the equations for solving the plate the terms

$$kx^2 + lxy \quad \text{and} \quad kxy + ly^2 \quad \text{in } x, y \text{ respectively}$$

where  $(k, l)$  are the required co-ordinates. Thus  $k$  is the same as  $\frac{r_1 - r_2}{2}$  or  $k \sin \psi$  in the previous notation.

Let  $a_1, a_2, a_3, a_4$  denote the mean residuals in  $x$  for each quadrant of the plate. Then if the reference stars are symmetrically grouped we have the four equations :

$$\begin{aligned} 6.5a + 6.5b + c + 42.25k + 42.25l &= a_1 \\ 6.5a - 6.5b + c + 42.25k - 42.25l &= a_2 \\ -6.5a + 6.5b + c + 42.25k - 42.25l &= a_3 \\ -6.5a - 6.5b + c + 42.25k + 42.25l &= a_4 \end{aligned}$$

and four similar equations in which  $d, e, f, l, k, \beta$  replace  $a, b, c, k, l, a$  respectively. From the equations we obtain

$$\begin{aligned} 169 \cdot l &= (a_4 - a_3) - (a_2 - a_1) \\ 169 \cdot k &= (\beta_4 - \beta_3) - (\beta_2 - \beta_1). \end{aligned}$$

If the plate is large the stars will be approximately symmetrically distributed, and the above coefficients 6.5, 42.25 will be nearly correct.

This method has been applied to some of the large Hyderabad plates, counts of which have been discussed above. The results are as follows :—

Plate No.	R.A.		$k$ .	$l$ .
	h	m		
405	7	0	+ '000007	- '000009
424	7	8	- '000010	+ '000002
402	7	16	- '000005	+ '000014
426	8	20	+ '000012	+ '000023

It is clear from these numbers that the measures are not affected by any appreciable tilt. If there is any tilt it is peculiar to each plate and may be treated as a small accidental error.

If the above numbers are correct, the Hyderabad plates may be considered free from any systematic tilt.

*Nizamiah Observatory, Hyderabad :*  
1915 April 22.

[*Note added June 20.*—If the plate used for testing the tilt with a collimator is not of uniform thickness, but is placed in the four different orientations in the telescope, the collimator remaining perpendicular to the back of the plate, the tilt will be correct, provided the centre of the O.G. appears to describe a circle about the cross-wires of the collimator as centre.]

*Measures of Double Stars.* By Eric Doolittle.

The double star measures of the Flower Astronomical Observatory which were principally made during the years 1907-1911 were published two months ago by the University of Pennsylvania, as vol. iv. part 1 of their Astronomical Series; and this volume contained measures on some 1954 different pairs. The following paper contains the mean results of the measures on 416 additional pairs, omitted from this volume. It will be noticed that a large proportion of these are wide and neglected H pairs, on which measures have been made in the course of the work of bringing the General Catalogue of Burnham up to date. Those which were not identified in the G.C. have been either so identified in Argelander's Durchmusterung, or, when too faint for this, their position has been determined by measurement from the nearest DM star. These identifications have been here added in the Notes, but for brevity the resulting right ascensions and declinations are omitted. Many hundreds of measures on the 8000 new stars, discovered since the publication of the G.C., are reserved for a later publication.

G.C.	Star.	Date.	p.	r.	Mags.		Nights.	Notes.
9893	H 1475	1907'54	273°1	7"09	10'4	11'5	3	1
9895	H 1478	1907'54	34'9	2'27	9'5	10'1	3	2
9895α	Anon	1907'54	222'2	15'69	9'4	9'5	2	3
9898	H 1473	1907'54	156'1	19'28	9'2	9'9	3	4
9899	H 902	1907'45	197'6	6'55	8'9	9'1	3	
9901	H 903	1907'50	353'3	10'95	10'3	11'2	3	5
9903	H 1479	1907'54	1'5	33'89	7'7	9'8	3	
9904	H 1476	1907'54	71'9	12'95	9'3	10'6	3	6
9911	H 904	1907'47	317'8	27'76	8'5	9'3	3	
9917	H 905	1907'47	171'3	11'65	8'8	10'5	3	
9918	H 2931	1907'65	313'1	6'50	10'1	10'8	3	AB } 7
		1907'69	209'8	14'08	...	12'7	2	AC }
9918α	Anon	1907'54	266'4	8'66	10'9	11'8	3	8
9920	H 2930	1909'52	156'4	28'37	10'7	11'2	2	9
9922	HCW 18	1905'79	314'8	21'21	8'0	8'7	2	
9925	Σ 2631	1907'02	339'6	4'65	...	...	3	
9927	A 278	1907'53	297'1	1'59	8'8	12'3	4	
9928	Hn 37	1911'52	308'8	3'10	9'0	11'3	3	
9930	Hu 80	1908'70	4'1	2'47	8'4	10'9	3	
9941	AG 247	1905'89	...	...	...	...	3	AB } 10
		1905'95	27'9	36'01	8'9	11'9	2	AC }
9942	Dawes 12	1907'56	100'5	28'26	9'5	11'9	3	11